Constructions

Lesson Synopsis:

In this lesson, students use compass and straightedge constructions to explore and make conjectures about attributes of geometric figures and their relationships. Students develop a deeper understanding of geometry by connecting the methods of construction to previous concepts in geometry. Procedural constructions are explored geometrically and in problem-solving settings. This lesson is introduced with the dilemma of determining a method for laying out a baseball field which engages the students and will ultimately serve as the evaluation piece.

TEKS:

G.2	***Geometric structure. The student analyzes geometric relationships in order to make and verify conjectures.***
G.2A	**Use constructions to explore attributes of geometric figures and to make conjectures about geometric relationships.**

Related TEKS:

G.3	***Geometric structure. The student constructs and justifies statements about geometric figures and their properties.***

GETTING READY FOR INSTRUCTION

Performance Indicator(s):

- Explore geometric properties using geometric constructions and attributes to create a geometric figure. Include a brief step-by-step summary of the construction and verification of the geometric relationships. (G.2A)
  ELPS ELPS: 1E, 1H, 2E, 2I, 3H, 4I, 5G

Key Understandings and Guiding Questions:

- Construction of geometric figures is based on the concepts of equidistance and linearity and application of geometric relationships.
  – Why are the compass and the straightedge the only tools used in geometric constructions?
  – What is required in order to construct a line, ray, or line segment?
  – How would a square and its diagonal be constructed from a given line segment?

Vocabulary of Instruction:

- constructions
- straightedge
- compass
- equidistant
- linearity

Materials:

- customary ruler
- calculator
- compass
- straightedge
- colored pencils

Resources:

- STATE RESOURCES
  – **Mathematics TEKS Toolkit:** Clarifying Activity/Lesson,/Assessments
  http://www.utdanacenter.org/mathtoolkit/index.php

  – **MTC Geometry:** Student Lessons – Investigation 1 Similarity, Explore/Explain, Act. 7 (Constructions)
  http://www.tea.state.tx.us/math/training/materials/Geometry/Documents/StudentLessons.pdf

  TMT^3 Geometry: Explore/Explain 2 (Geometric Properties and Sketchpad Skills), Elaborate, (Ring Around the Rose Window)
  http://www.tea.state.tx.us/math/training/materials/TMT3/index.htm

Advance Preparation:

1. Handout: **Play Ball! (Almost)** (1 per student)
2. Handout: **What Is a Construction?** (1 per student)
3. Handout: **Geometric Constructions** (1 per student)
4. Handout: **Geometric Constructions, Part 2** (1 per student)
5. Handout: **Applications of Constructions** (1 per student)
6. Handout: **Take Me Out to the Ballpark!** (1 per student)

Background Information:

Although geometric constructions are presented in isolation from other units of instruction and only basic procedural knowledge is needed to duplicate constructions, students will benefit from concepts and skills developed throughout geometry. For example, an understanding of congruent triangle relationships can aid the student in understanding and justifying the method for constructing the perpendicular bisector of a segment; understanding the ratios of special right triangles can make easy work of constructing special right triangles without relying on constructing each angle of the triangle; and an understanding of parallel lines and transversals can aid students in understating and justifying several other constructions. While not essential for teaching students to do constructions, this background knowledge of geometric concepts as well as others will aid students in justifying the methods of constructions and developing an appreciation for their use.

GETTING READY FOR INSTRUCTION SUPPLEMENTAL PLANNING DOCUMENT

Instructors are encouraged to supplement, and substitute resources, materials, and activities to differentiate instruction to address the needs of learners. The Exemplar Lessons are one approach to teaching and reaching the Performance Indicators and Specificity in the Instructional Focus Document for this unit. A Microsoft Word template for this planning document is located at www.cscope.us/sup_plan_temp.doc. If a supplement is created electronically, users are encouraged to upload the document to their Lesson Plans as a Lesson Plan Resource in your district Curriculum Developer site for future reference.

INSTRUCTIONAL PROCEDURES

Instructional Procedures

ENGAGE

1. Distribute the handout: **Play Ball! (Almost)** to each student.
2. Direct students to read through the passage and discuss in whole group.
3. Have students work in pairs and complete the four questions. Make sure each student has a ruler and calculator. Make sure each group has colored pencils.
4. When most students have finished the activity, have volunteers display their scale models of the baseball infield.
5. Facilitate a discussion of the method described for laying out a baseball diamond.
   Facilitation questions:
   - **Why do you suppose this method for laying out a baseball field works? Can you support your answer with geometry?** *This method is based on the isosceles right triangle where the distance between home plate and second base is the length of the hypotenuse. Solving the isosceles right triangle for the length of a leg given a hypotenuse of 127 feet, 3 and* $\frac{3}{8}$ *inches shows that each leg is very close to 90 feet. Swinging the tapes and finding their intersection at ninety feet locates the vertex of a right angle which corresponds to the corner of first or third base.*
   - **How could you use properties of geometry to verify that the resulting layout is indeed a square and therefore correct?** *Once the baseball diamond is laid out, measuring to see that the diagonals are the same length will ensure that the figure is a rectangle and*

Notes for Teacher

NOTE: 1 Day = 50 minutes
Suggested Day 1

MATERIALS

- Handout: **Play Ball! (Almost)** (1 per student)
- customary ruler
- colored pencils
- calculator

TEACHER NOTE
The purpose of the ENGAGE is to generate interest in the geometric properties used in setting up a baseball field and to set the stage for geometric constructions. The method of laying out a baseball field discussed makes use of a technique (finding the intersection of the swinging tape measures) that is very similar to finding the intersection of compass arcs used in geometric constructions.

STATE RESOURCES
Mathematics TEKS Toolkit: Clarifying Activity/Lesson/Assessment may be used to reinforce these concepts or

Instructional Procedures

therefore contains right angles; the fact that each base line is 90 feet ensures that the figure is a rhombus. Since the figure is both a rectangle and a rhombus, it is also a square.

- **Suppose you did not have any measuring devices (tape measures, protractors, etc.) but you did have a length of rope that represented the distance between the bases. Could you use this arbitrary length of rope to lay out a baseball diamond? Why or why not?** *At this point students are most likely to respond that it would not be possible. However, the purpose of this question is to set the stage for geometric constructions which do not use conventional measures.*

Notes for Teacher

used as alternate activities.

EXPLORE/EXPLAIN

Day 2

1. Distribute the handout: **What Is a Construction?** to each student. Formally introduce the concept of a geometric construction using the information from the handout.
2. Have each student read the introductory paragraph silently, then discuss it with a partner. Have partners share out their definitions of constructions in whole group discussion.
3. In whole group discussion go over the non-construction examples and discuss why they do not meet the correct definition of a construction.
4. Distribute the handout: **Geometric Constructions** to each student.
5. Have students work in pairs to complete the construction activities. Each student should complete their own constructions, however, students will benefit by collaborating with others to justify their conjectures.

Day 3

6. Debrief the previous handout, **Geometric Constructions**, with appropriate questions in whole group discussion.
   Facilitation questions:
   - **In the first activity, what appears to be true about the two segments? How can you verify your conjecture?** *They have the same length. They are congruent. This can be verified by tracing or measuring.*
   - **In the second activity, what appears to be true about $\angle A$ and $\angle FDE$? How can you verify your conjecture?** *The angles appear to be congruent or have the same measure. This can be verified by tracing or measuring.*
   - **In the third activity, what appears to be true about $\overrightarrow{PW}$? How can you verify your conjecture?** $\overleftrightarrow{PW}$ *appears to bisect the angle. This can be verified by measuring or tracing each of the smaller angles, or paper-folding.*
   - **In the fourth activity, what appears to be true about point *M*? What appears to be true about $\overleftrightarrow{AB}$ and $\overline{XY}$? How can you verify your previous conjectures?** *Point M appears to be the midpoint. The two lines appear to be perpendicular. The conjectures can be verified by measuring or paper-folding.*
   - **In the fifth activity, what appears to be true about $\overleftrightarrow{AP}$ and line *k*? How can you verify your conjecture?** *They appear to be perpendicular. This can be verified by folding the paper along either line thus showing two adjacent congruent angles which imply the angles are right angles.*
   - **In the sixth activity, what appears to be true $\overleftrightarrow{BR}$ and line *k*? How can you verify your conjecture?** *They appear to be perpendicular.*

Suggested Days 2-3

MATERIALS

- Handout: **What Is a Construction?** (1 per student)
- Handout: **Geometric Constructions** (1 per student)
- Handout: **Geometric Constructions, Part 2** (1 per student)
- compass
- straightedge

TEACHER NOTE

The purpose of this activity is for students to define and practice constructions as well as develop an understanding of why they are possible by justifying conjectures based on their previous knowledge of geometric properties.

TEACHER NOTE

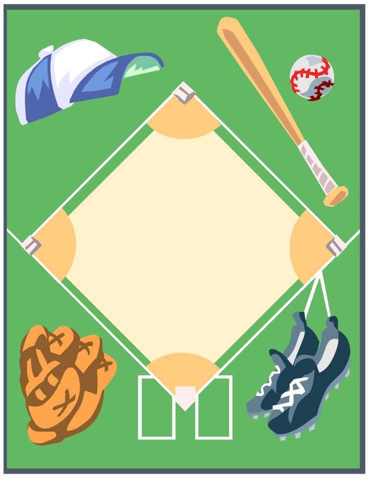
When formally introducing the concept of geometric constructions, be sure to emphasize to students that the only allowable tools used in constructions are the straightedge and the compass. Be sure to discuss examples of constructions as well as non-examples with an emphasis on what makes a good construction; for example, constructions lines are drawn with the compass lightly to mark locations while lines that are part of the figure are drawn darker, etc.

Instructional Procedures	Notes for Teacher
This can be verified by folding the paper along either line thus showing two adjacent congruent angles which imply the angles are right angles. • **Can you explain how the compass and straightedge can be used as unconventional tools for measurement?** *The straightedge provides linearity or a direction for measurement while the compass provides equidistance since the distance or measure between the two legs of the compass is constant for a given arc or circle.* 7. Distribute copies of handout: **Geometric Constructions, Part 2** to each student. Have students complete the construction problems for practice.	
ELABORATE	**Suggested Days 4-5 (2 days)**
1. Distribute copies of handout: **Applications of Constructions** to each student. 2. Have students complete the application problems using constructions.	**MATERIALS** • Handout: **Applications of Constructions** (1 per student) • compass • straightedge **TEACHER NOTE** The purpose of the ELABORATE is to allow students to use constructions in a problem-solving setting. Constructions are used to solve real-world problems that might otherwise be difficult to solve using traditional methods involving measurement and calculation. **STATE RESOURCES** **MTC Geometry:** Student Lessons – Investigation 1 Similarity, Explore/Explain, Act. 7 (Constructions) may be used to reinforce these concepts or used as alternate activities. **TMT^3 Geometry:** Explore/Explain 2 (Geometric Properties and Sketchpad Skills), Elaborate, (Ring Around the Rose Window) may be used to reinforce these concepts or used as alternate activities.
EVALUATE	**Suggested Day 6**
1. Distribute copies of handout: **Take Me Out to the Ballpark!** to each student. 2. Have students complete the activity in order to assess their understanding of constructions.	**MATERIALS** • Handout: **Take Me Out to the Ballpark!** (1 per student) • compass • straightedge **TEACHER NOTE** The purpose of the EVALUATE is to give students the opportunity to demonstrate their understanding of geometric constructions. The scenario from the ENGAGE is revisited and students are challenged to create a layout for a baseball field using

Instructional Procedures	Notes for Teacher
	geometric constructions. Because there are various ways a student can accomplish the layout, it is important that the student provide a step-by-step procedure for their product. Teachers should use the students' products as well as their step-by-step procedures as a basis for evaluating understanding.

Play Ball! (Almost) KEY

Laying out a baseball field for play normally involves locating a line through home plate to second base and measuring a distance of 127 feet $3\frac{3}{8}$ inches from the tip of home plate to the *center* of second base. Once these two bases are located, tape measures are anchored at home plate and the *center* of second base. The tape measures are extended in the direction of first base. The point at which the tapes intersect at 90 feet represents the back corner of first base. Third base is found in a similar manner. Essentially the baseball diamond is a square with edge length 90 feet. First and third bases fit within the square while the *center* of second base is placed at one of the vertices of the square.



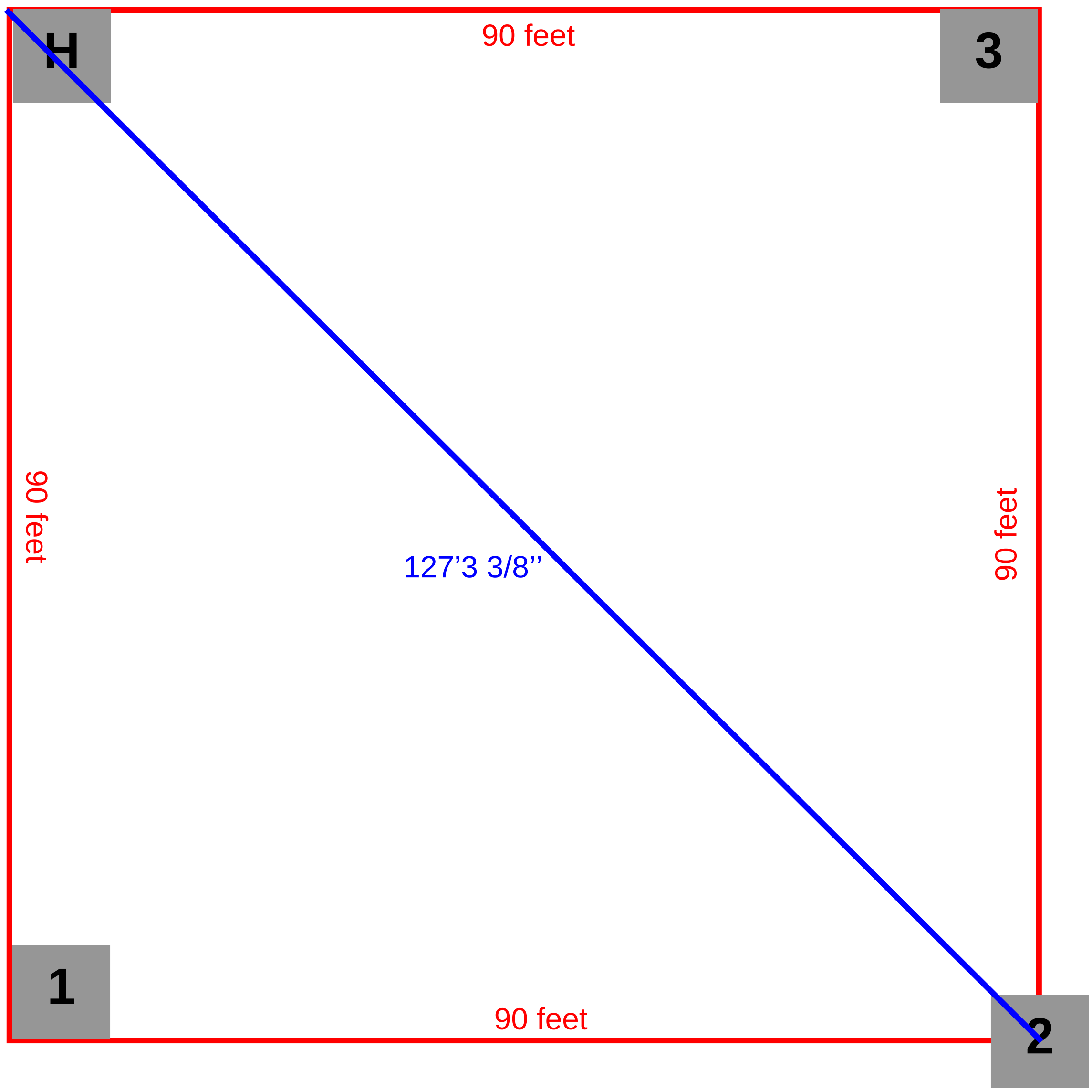
1. On a separate sheet of paper or on the back of this page, use the directions above to construct a scale model of a baseball infield. Let $\frac{1}{16}$ inch represent 1 foot. Mark and label the measured distances in color.
   See diagram on next page.

2. Why do you suppose this method works? Can you support your answer with geometry?
   This method is based on the isosceles right triangle where the distance between home plate and second base is the length of the hypotenuse. Solving the isosceles right triangle for the length of a leg given a hypotenuse of 127 feet, $3\frac{3}{8}$ inches shows that each leg is very close to 90 feet. Swinging the tapes and finding their intersection at ninety feet locates the vertex of a right angle which corresponds to the corner of first or third base.

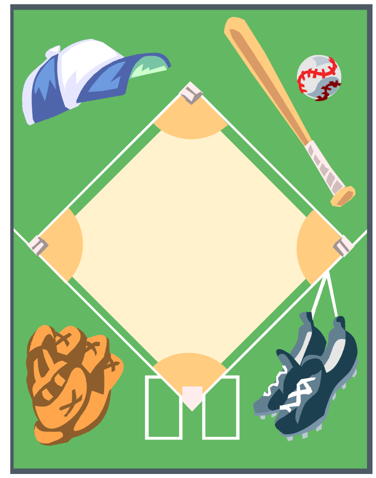
3. How could you use properties of geometry to verify that the resulting layout is indeed a square and therefore correct?
   Once the baseball diamond is laid out, measuring to see that the diagonals are the same length will ensure that the figure is a rectangle and therefore contains right angles; the fact that each base line is 90 feet ensures that the figure is a rhombus. Since the figure is both a rectangle and a rhombus, it is also a square.

4. Suppose you did not have any measuring devices (tape measures, protractors, etc.) but you did have a length of rope that represented the distance between the bases. Could you use this arbitrary length of rope to lay out a baseball diamond? Why or why not?
   At this point students are most likely to respond that it would not be possible. However, the purpose of this question is to set the stage for geometric constructions which do not use conventional measures.

Problem #1 Diagram KEY



Play Ball! (Almost)



Laying out a baseball field for play normally involves locating a line through home plate to second base and measuring a distance of 127 feet $3\frac{3}{8}$ inches from the tip of home plate to the *center* of second base. Once these two bases are located, tape measures are anchored at home plate and the *center* of second base. The tape measures are extended in the direction of first base. The point at which the tapes intersect at 90 feet represents the back corner of first base. Third base is found in a similar manner. Essentially the baseball diamond is a square with edge length 90 feet. First and third bases fit within the square while the *center* of second base is placed at one of the vertices of the square.

1. Use the directions introduction to construct a scale model of a baseball infield. Let $\frac{1}{16}$ inch represent 1 foot. Mark and label the measured distances in color.

2. Why do you suppose this method works? Can you support your answer with geometry?

3. How could you use properties of geometry to verify that the resulting layout is indeed a square and therefore correct?

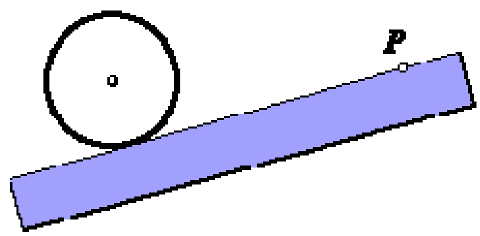
4. Suppose you did not have any measuring devices (tape measures, protractors, etc.) but you did have a length of rope that represented the distance between the bases. Could you use this arbitrary length of rope to lay out a baseball diamond? Why or why not?

What Is a Construction? (pp. 1 of 2)

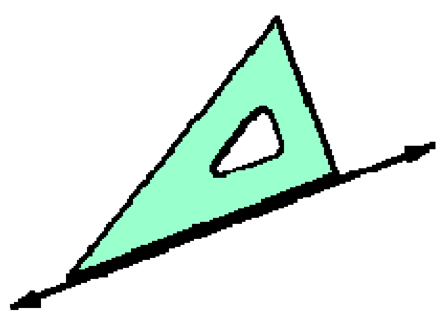
Constructions:
Geometric constructions go back to Greek antiquity. They are often called Euclidean constructions, but they actually predate Euclid. Geometric constructions are made using only a compass and straightedge. The compass establishes equidistance and the straightedge establishes linearity. All geometric constructions are based on those two concepts.
The compass is anchored at a center point and keeps the pencil at a fixed distance from that point. Therefore, all points on the curve drawn by a compass are equidistant from the center point.
In constructions, rulers are used as straightedges, and the graduation marks are not used. The straightedge is used only for drawing lines. Given any two distinct points, this instrument can draw the set of all points that are collinear with them. No measurements are allowed in constructions!

Non-Constructions:
Using construction instruments is not sufficient for making your drawing a construction. Here are some guidelines and common mistakes.

To construct a line, a ray, or a line segment, the straightedge must be aligned across exactly two fixed points. In this figure, the objective is to construct a line through point *P*, tangent to the circle. The straightedge has been placed on point *P*, and is being rotated into position. This is not a construction. The direction of the line cannot be established by approximating tangency. Another point must be constructed.

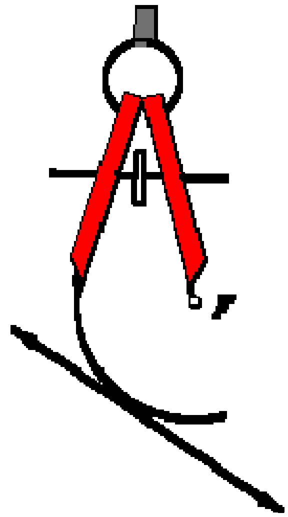


Drafting triangles are often used as straightedges. This practice is acceptable, but do not use the angles as templates. In this figure, a drafting triangle is being used to draw a perpendicular line. This is not a construction. In essence, the instrument is being used to measure a 90° angle, just as a protractor would be used.

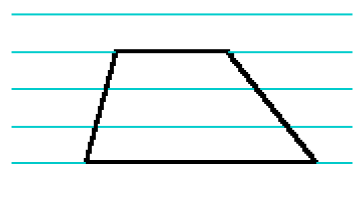


What Is a Construction? (pp. 2 of 2)

Here is another misguided attempt to approximate tangency. This time it is with a compass. When constructing curves (other than arbitrary curves), the compass must have a fixed center point, and the radius must be set by the distance between two fixed points. In the figure at right, the radius is only a guess. Another point must be constructed to set the radius.



Ruled paper can be convenient for sketching, but do not mistake this for a construction. The trapezoid at the right is not constructed but sketched by using the ruled paper. On this same note, do not lay a ruler down and draw along both edges in order to make parallel lines. That is not construction either.

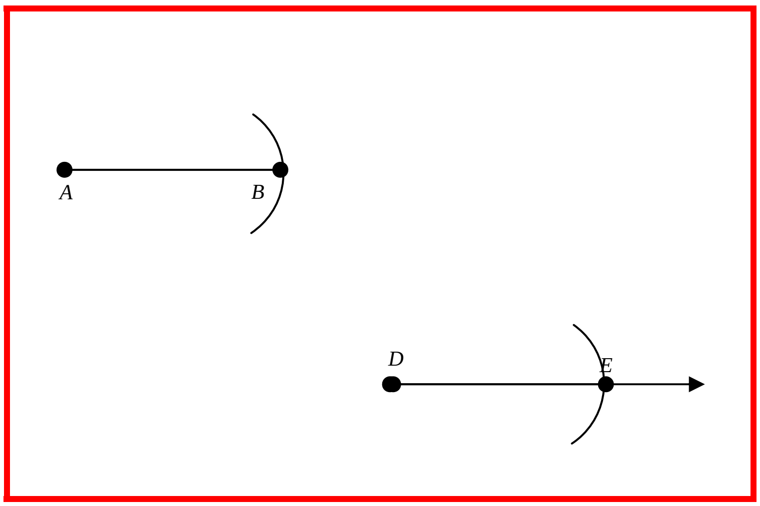


Geometric Constructions (pp. 1 of 6) KEY

Previously, you were asked if it was possible to lay out a square of given side length without the aid of any conventional measuring devices such as a tape measure or protractor. Before attempting to fully answer that question, let's turn our attention to how constructions are used for measuring.

1. Using a straightedge, lightly draw a line that is somewhat longer than $\overline{AB}$ pictured below.
   a. Locate and label point D toward either end of your drawing.
   b. Open your compass to a setting so that the point of your compass is at A and the pencil of your compass is at B. Place your compass on A and draw an arc across the endpoint B.
   c. Without changing the compass radius, place the compass on D and draw an arc crossing the ray. Label the intersection point as E. Draw $\overline{DE}$.

Sample answer:



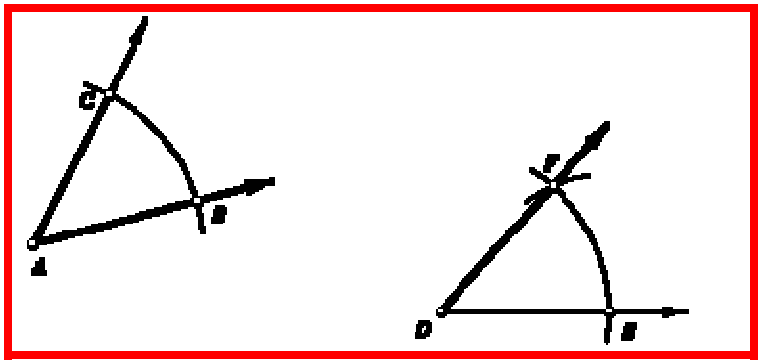
   d. What appears to be true about the two segments? How can you verify your conjecture?

They have the same length. They are congruent. This can be verified by tracing or measuring.

Geometric Constructions (pp. 2 of 6) KEY

2. Given $\angle A$ and the ray with endpoint *D* below, place your compass on *A* and draw an arc across both sides of the angle.
   a. Label the points of intersection of the arc and the angle as *C* and *B*.
   b. Without changing the compass radius, place the compass on *D* and draw a long arc crossing the ray. Label the intersection of the arc and the ray as *E*.
   c. Set the compass so that its radius is *BC*. Place the compass on *E* and draw an arc intersecting the one drawn in the previous step. Label the intersection point *F*.
   d. Use the straightedge to draw $\overrightarrow{DF}$.

Sample answer:



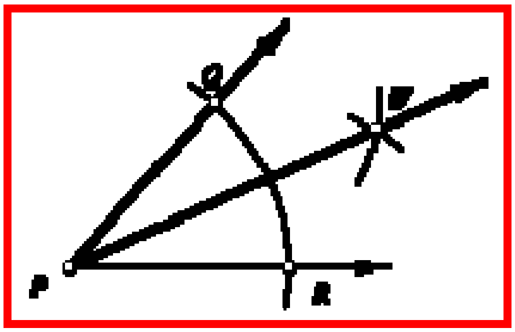
   e. What appears to be true about $\angle A$ and $\angle FDE$? How can you verify your conjecture?

   The angles appear to be congruent or have the same measure. This can be verified by tracing or measuring.

Geometric Constructions (pp. 3 of 6) KEY

3. Given the angle below, place the compass on *P* and draw an arc across both sides of the angle. Label the intersection points *Q* and *R*.
   a. Place the compass on *Q* and draw an arc across the interior of the angle.
   b. Without changing the radius of the compass, place it on *R* and draw an arc intersecting the one drawn in the previous step. Label the intersection point *W*.
   c. Using the straightedge, draw $\overrightarrow{PW}$.

Sample answer:



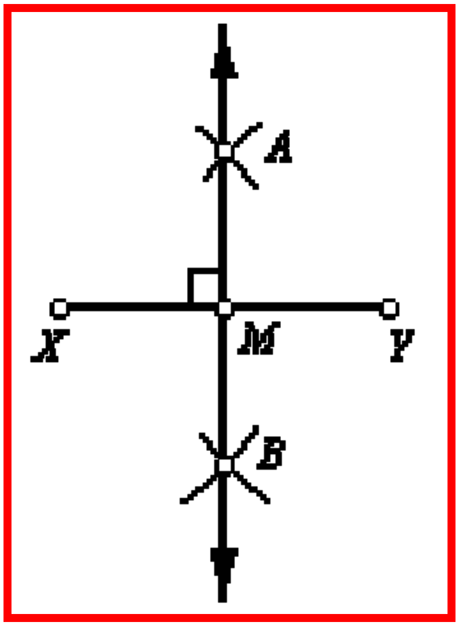
   d. What appears to be true about $\overrightarrow{PW}$? How can you verify your conjecture?

   $\overrightarrow{PW}$ appears to bisect the angle. This can be verified by measuring or tracing each of the smaller angles, or paper-folding.

Geometric Constructions (pp. 4 of 6) KEY

4. Given $\overline{XY}$, place the compass at *X*. Adjust the compass radius so that it is more than $\frac{1}{2}\overline{XY}$.
   a. Without changing settings of the compass, place the compass at each end of $\overline{XY}$ and draw arcs that intersect both above and below $\overline{XY}$. Label the intersection points *A* and *B*.
   b. Using the straightedge, draw $\overleftrightarrow{AB}$ so that it intersects $\overline{XY}$. Label the intersection point *M*.

Sample answer:



   c. What appears to be true about *M*?

   Point M appears to be the midpoint.

   d. What appears to be true about $\overleftrightarrow{AB}$ and $\overline{XY}$?

   The two lines appear to be perpendicular.

   e. How can you verify your conjectures from parts *c* and *d*?

   The conjectures can be verified by measuring or paper-folding.

Geometric Constructions (pp. 5 of 6) KEY

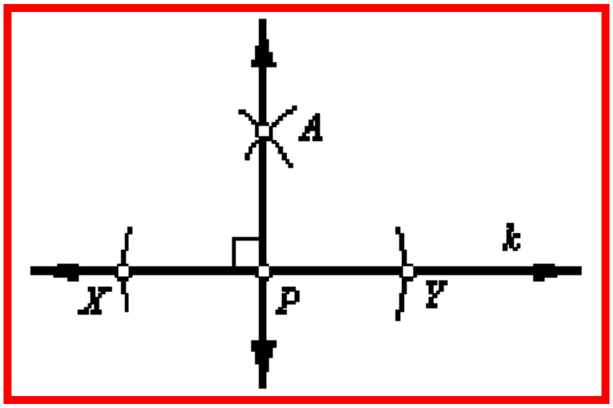
5. Given line *k* with point *P*, place the compass on *P*. Using an arbitrary radius, draw arcs intersecting line *k* at two points either side of *P*. Label the intersection points *X* and *Y*.

   a. Place the compass at *X*. Adjust the compass radius so that it is more than $\frac{1}{2}\overline{XY}$.

   b. Without changing the setting of the compass, draw arcs from *X* and *Y* that intersect above line *k*. Label the point of intersection of the arcs as *A*.

   c. Using a straightedge, draw $\overleftrightarrow{AP}$.

Sample answer:



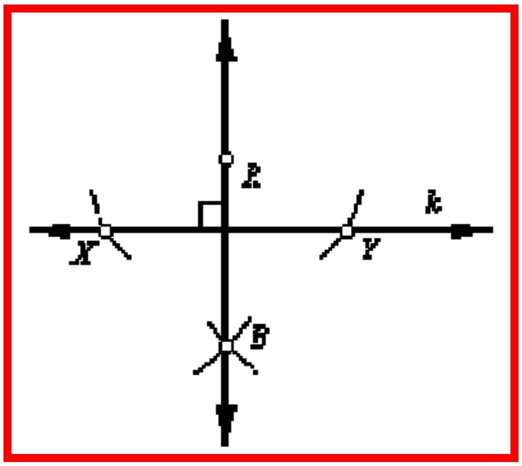
   d. What appears to be true about $\overleftrightarrow{AP}$ and line *k*? How can you verify your conjecture?

   They appear to be perpendicular. This can be verified by folding the paper along either line thus showing two adjacent congruent angles which imply the angles are right angles.

Geometric Constructions (pp. 6 of 6) KEY

6. Given line *k* and point *R* not on line *k*, place the compass on *R*. Using an arbitrary radius, draw arcs intersecting line *k* at two points. Label the intersection points *X* and *Y*.
   a. Place the compass at *X*. Adjust the compass radius so that it is more than $\frac{1}{2}\overline{XY}$.
   b. Without changing the setting of the compass, draw arcs from *X* and *Y* that intersect below line *k*. Label the point of intersection of the arcs as *B*.
   c. Using a straightedge, draw $\overleftrightarrow{BR}$.

Sample answer:



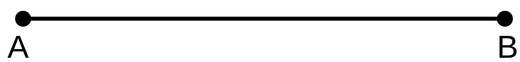
   d. What appears to be true about $\overleftrightarrow{BR}$ and line *k*? How can you verify your conjecture?

   They appear to be perpendicular. This can be verified by folding the paper along either line thus showing two adjacent congruent angles which imply the angles are right angles.

Geometric Constructions **(pp. 1 of 6)**

Previously, you were asked if it was possible to lay out a square of given side length without the aid of any conventional measuring devices such as a tape measure or protractor. Before attempting to fully answer that question, let's turn our attention to how constructions are used for measuring.

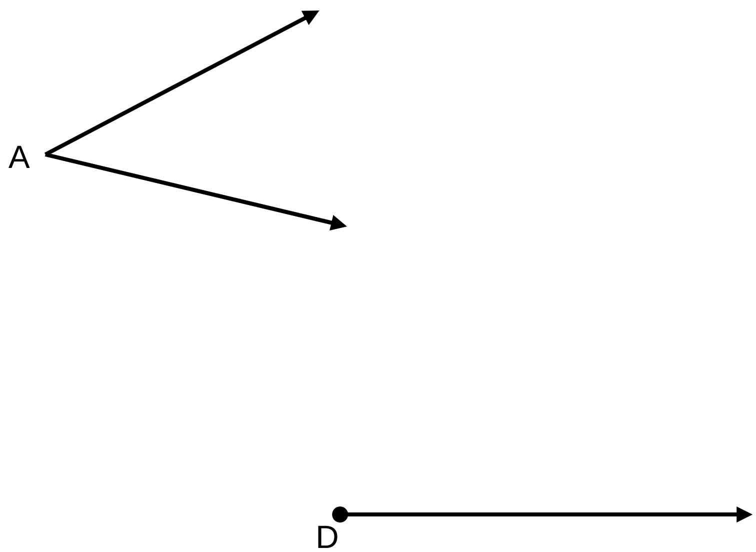
1. Using a straightedge, lightly draw a line that is somewhat longer than $\overline{AB}$ pictured below.
   a. Locate and label point D toward either end of your drawing.
   b. Open your compass to a setting so that the point of your compass is at A and the pencil of your compass is at B. Place your compass on A and draw an arc across the endpoint B.
   c. Without changing the compass radius, place the compass on D and draw an arc crossing the ray. Label the intersection point as E. Draw $\overline{DE}$.



   d. What appears to be true about the two segments? How can you verify your conjecture?

Geometric Constructions (pp. 2 of 6)

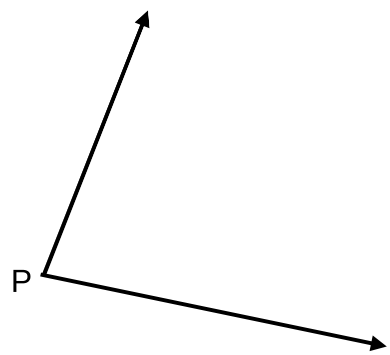
2. Given $\angle A$ and the ray with endpoint *D* below, place your compass on *A* and draw an arc across both sides of the angle.
   a. Label the points of intersection of the arc and the angle as *C* and *B*.
   b. Without changing the compass radius, place the compass on *D* and draw a long arc crossing the ray. Label the intersection of the arc and the ray as *E*.
   c. Set the compass so that its radius is *BC*. Place the compass on *E* and draw an arc intersecting the one drawn in the previous step. Label the intersection point *F*.
   d. Use the straightedge to draw $\overrightarrow{DF}$.



   e. What appears to be true about $\angle A$ and $\angle FDE$? How can you verify your conjecture?

Geometric Constructions **(pp. 3 of 6)**

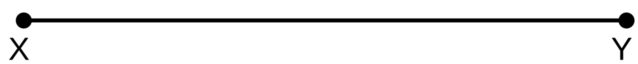
3. Given the angle below, place the compass on *P* and draw an arc across both sides of the angle. Label the intersection points *Q* and *R*.
   a. Place the compass on *Q* and draw an arc across the interior of the angle.
   b. Without changing the radius of the compass, place it on *R* and draw an arc intersecting the one drawn in the previous step. Label the intersection point *W*.
   c. Using the straightedge, draw $\overrightarrow{PW}$.



   d. What appears to be true about $\overrightarrow{PW}$? How can you verify your conjecture?

Geometric Constructions **(pp. 4 of 6)**

4. Given $\overline{XY}$, place the compass at *X*. Adjust the compass radius so that it is more than $\frac{1}{2}\overline{XY}$.
   a. Without changing settings of the compass, place the compass at each end of $\overline{XY}$ and draw arcs that intersect both above and below $\overline{XY}$. Label the intersection points *A* and *B*.
   b. Using the straightedge, draw $\overleftrightarrow{AB}$ so that it intersects $\overline{XY}$. Label the intersection point *M*.



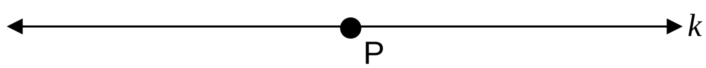
   c. What appears to be true about *M*?

   d. What appears to be true about $\overleftrightarrow{AB}$ and $\overline{XY}$?

   e. How can you verify your conjectures from parts *c* and *d*?

Geometric Constructions **(pp. 5 of 6)**

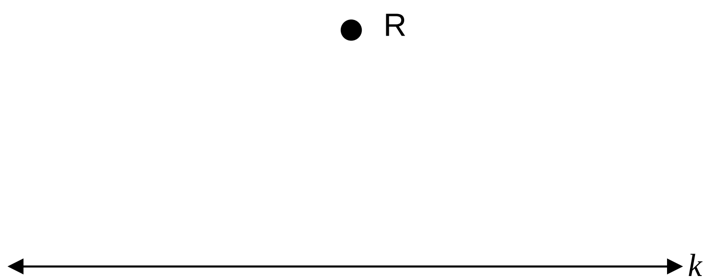
5. Given line *k* with point *P*, place the compass on *P*. Using an arbitrary radius, draw arcs intersecting line *k* at two points either side of *P*. Label the intersection points *X* and *Y*.
   a. Place the compass at *X*. Adjust the compass radius so that it is more than $\frac{1}{2}\overline{XY}$.
   b. Without changing the setting of the compass, draw arcs from *X* and *Y* that intersect above line *k*. Label the point of intersection of the arcs as *A*.
   c. Using a straightedge, draw $\overleftrightarrow{AP}$.



   d. What appears to be true about $\overleftrightarrow{AP}$ and line *k*? How can you verify your conjecture?

Geometric Constructions (pp. 6 of 6)

6. Given line k and point R not on line k, place the compass on R. Using an arbitrary radius, draw arcs intersecting line k at two points. Label the intersection points X and Y.
   a. Place the compass at X. Adjust the compass radius so that it is more than $\frac{1}{2}\overline{XY}$.
   b. Without changing the setting of the compass, draw arcs from X and Y that intersect below line k. Label the point of intersection of the arcs as B.
   c. Using a straightedge, draw $\overleftrightarrow{BR}$.

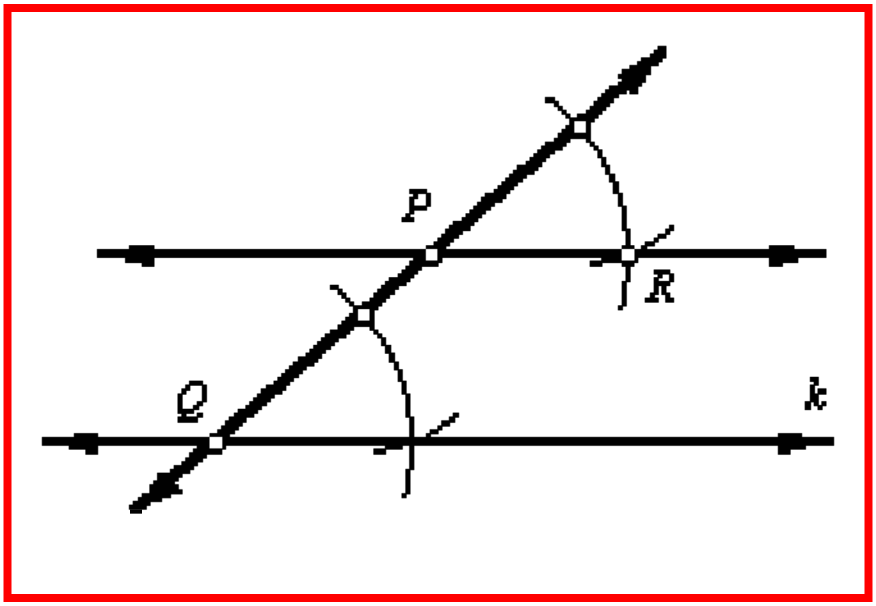


   d. What appears to be true about $\overleftrightarrow{BR}$ and line k? How can you verify your conjecture?

Geometric Constructions, Part 2 (pp. 1 of 7) KEY

1. Given a line and a point, construct a line through the point, parallel to the given line.
   a. Begin by drawing a point *P* and line *k*, with *P* not on line *k*.
   b. Draw an arbitrary line through *P*, intersecting line *k*. Call the intersection point *Q*. Now the task is to construct an angle with vertex *P*, congruent to the angle of intersection.
   c. Center the compass at *Q* and draw an arc intersecting both lines. Without changing the radius of the compass, center it at *P* and draw another arc.
   d. Set the compass radius to the distance between the two intersection points of the first arc. Now center the compass at the point where the second arc intersects $\overleftrightarrow{PQ}$. Mark the arc intersection point *R*.

Sample answer:



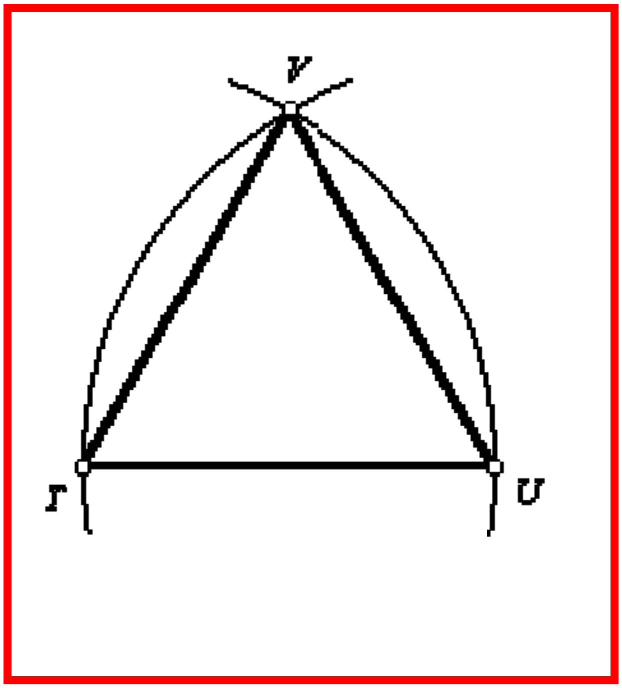
   e. Explain how you know that $\overleftrightarrow{PR}$ is parallel to line *k*.

The conjectures can be verified by measuring or paper-folding.

Geometric Constructions, Part 2 (pp. 2 of 7) KEY

2. Given a line segment as one side, construct an equilateral triangle. This method may also be used to construct a 60^o angle.
   a. Begin with $\overline{TU}$.
   b. Center the compass at *T*, and set the compass radius to $\overline{TU}$. Draw an arc as shown.
   c. Keeping the same radius, center the compass at *U* and draw another arc intersecting the first one. Let *V* be the point of intersection.
   d. Draw $\overline{TV}$ and $\overline{UV}$.

Sample answer:

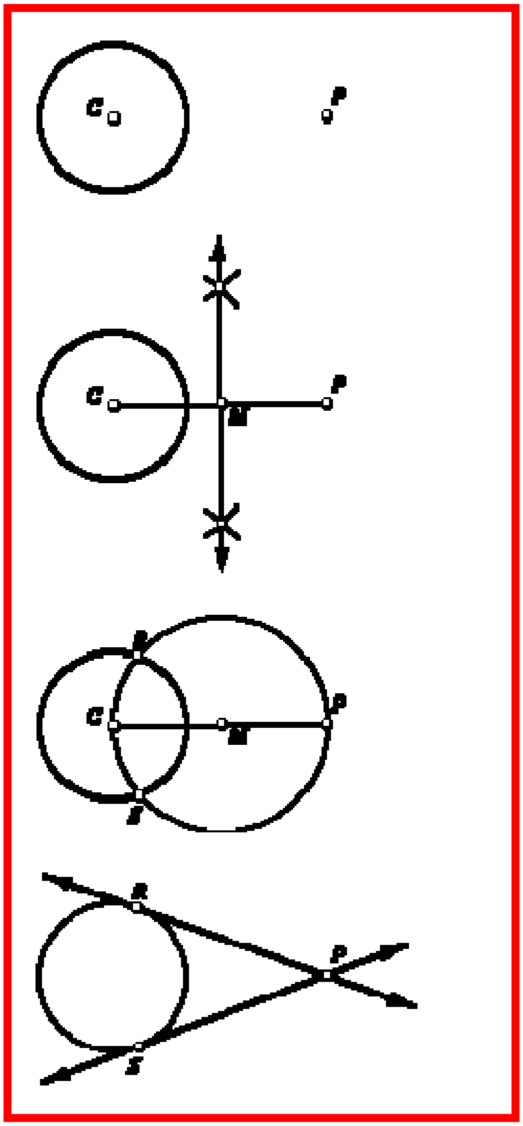


   e. Explain how you know the triangle is an equilateral triangle, and each of its interior angles has a measure of 60°. *Since the same radius, TU, was used to construct to the intersecting point V from point T and point U, the sides of the triangle were all equal in length. Therefore, the triangle is called equilateral. All angles in an equilateral triangle measure 60°. Or, after measuring, I found the sides were congruent in length and the angles were all 60°.*

Geometric Constructions, Part 2 (pp. 3 of 7) KEY

3. Given a circle, its center point, and a point on the exterior of the circle, constructs a line through the exterior point, tangent to the circle.
   a. Begin with a circle with center at *C*.
   b. *P* is on the exterior of the circle.
   c. Draw line segment $\overline{CP}$, and construct *M*, the midpoint of $\overline{CP}$. **Refer to the perpendicular bisector construction for the construction of the midpoint.**
   d. Center the compass on *M*. Draw a circle through *C* and *P*. It will intersect the other circle at two points, *R* and *S*.
   e. *R* and *S* are the tangent points. $\overleftrightarrow{PR}$ and $\overleftrightarrow{PS}$ are tangent to the circle centered on *C*.

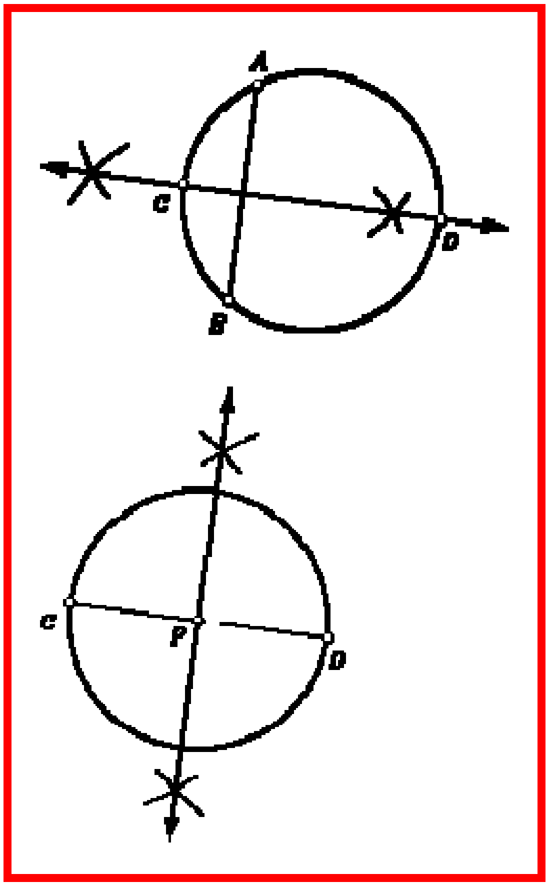
Sample answer:



Geometric Constructions, Part 2 (pp. 4 of 7) KEY

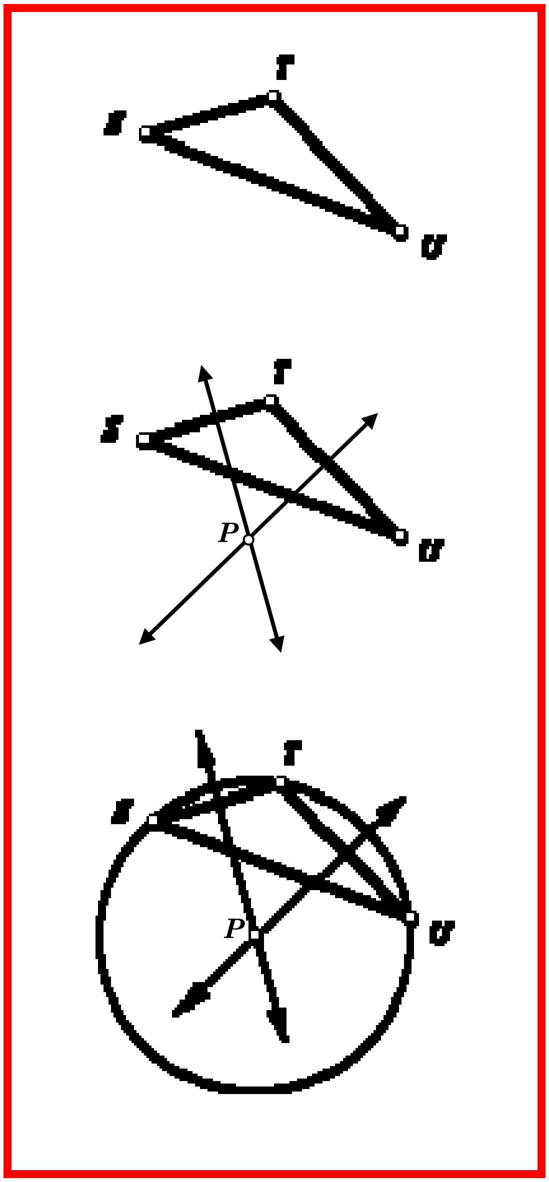
4. Construct the center point of a given circle.
   a. Begin with a circle, but no center point.
   b. Draw chord $\overline{AB}$.
   c. Construct the perpendicular bisector of chord $\overline{AB}$. Let *C* and *D* be the points where it intersects the circle. **Refer to the construction of a perpendicular bisector.**
   d. Chord $\overline{CD}$ is a diameter of the circle. Construct *P*, the midpoint of diameter $\overline{CD}$. *P* is the center point of the circle. **Refer to the construction of a perpendicular bisector.**

Sample answer:



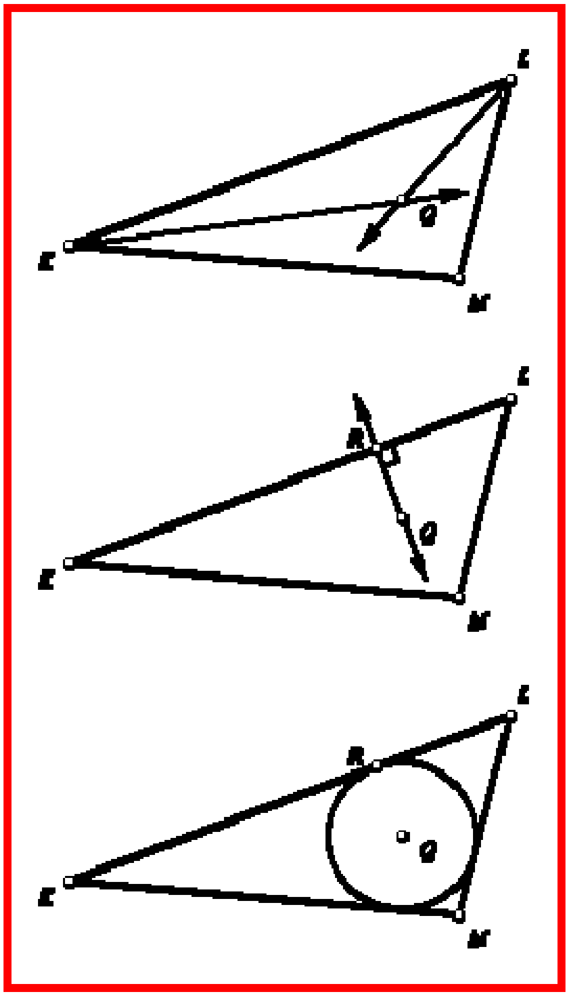
Geometric Constructions, Part 2 (pp. 5 of 7) KEY

5. Given a triangle, circumscribe a circle.
   a. Begin with ΔSTU.
   b. Construct the perpendicular bisectors of $\overline{ST}$ and $\overline{TU}$. **Refer to the construction of a perpendicular bisector.** Label the point of intersection as *P*.
   c. Center the compass on *P*, and draw the circle through *S*, *T*, and *U*.



Geometric Constructions, Part 2 (pp. 6 of 7) KEY

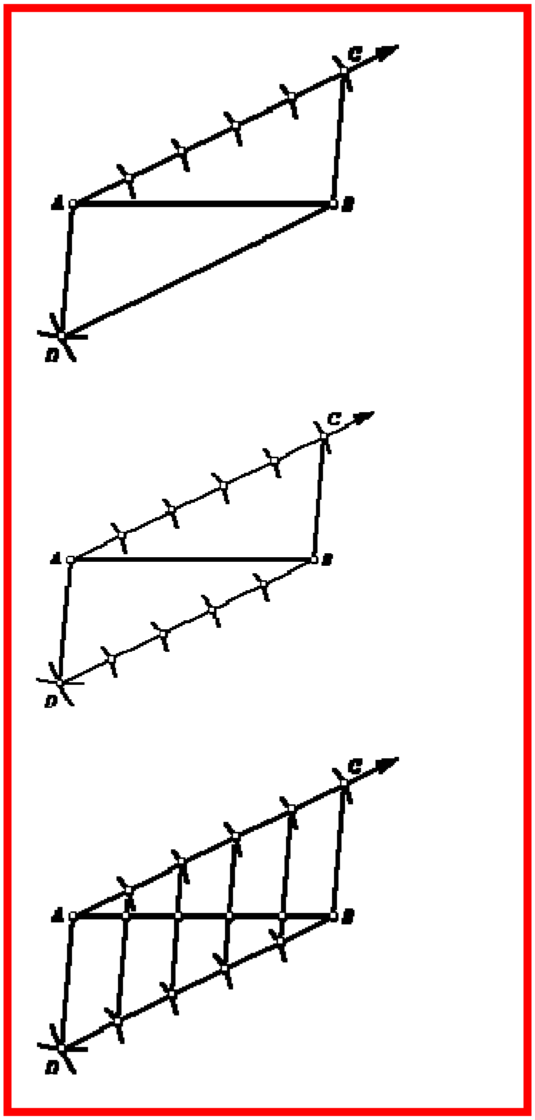
6. Given a triangle, inscribe a circle.
   a. Begin with ΔKLM .
   b. Construct the bisectors of $\angle K$ and $\angle L$. Refer **to the construction of an angle bisector.** Let *Q* be the intersection of the two angle bisectors.
   c. Construct a line through *Q*, perpendicular to $\overline{KL}$. Let *R* be the point of intersection. **Refer to the construction of a perpendicular through a given point.**
   d. Center the compass on *Q*, and draw a circle through *R*. The circle will be tangent to all three sides of a triangle.
   Sample answer:



Geometric Constructions, Part 2 (pp. 7 of 7) KEY

7. Divide a line segment into n congruent line segments. In this example, n = 5.
   a. Begin with $\overline{AB}$. It will be divided into five congruent line segments.
   b. Draw a ray from *A*. Use the compass to step off five uniformly spaced points along the ray. Label the last intersection point *C*.
   c. Draw an arc with the compass centered at point *A*, with radius *BC*. Draw a second arc with the compass centered at point *B*, with radius *AC*. Label the intersection point *D*. Note that *ACBD* is a parallelogram.
   d. Use the compass to step off points along $\overline{DB}$, using the same radius that was used for the points along $\overline{AC}$.
   e. Use the straightedge to connect the corresponding points. These line segments will be parallel. They cut $\overline{AC}$ and $\overline{DB}$ into congruent segments. Therefore, they must also cut $\overline{AB}$ into congruent segments.

Sample answer:



Geometric Constructions, Part 2 (pp. 1 of 7)

1. Given a line and a point, construct a line through the point, parallel to the given line.
   a. Begin by drawing a point *P* and line *k*, with *P* not on line *k*.
   b. Draw an arbitrary line through *P*, intersecting line *k*. Call the intersection point *Q*. Now the task is to construct an angle with vertex *P*, congruent to the angle of intersection.
   c. Center the compass at *Q* and draw an arc intersecting both lines. Without changing the radius of the compass, center it at *P* and draw another arc.
   d. Set the compass radius to the distance between the two intersection points of the first arc. Now center the compass at the point where the second arc intersects $\overleftrightarrow{PQ}$. Mark the arc intersection point *R*.

   e. Explain how you know that $\overleftrightarrow{PR}$ is parallel to line *k*.

Geometric Constructions, Part 2 (pp. 2 of 7)

2. Given a line segment as one side, construct an equilateral triangle. This method may also be used to construct a 60^{o} angle.
   a. Begin by drawing $\overline{TU}$.
   b. Center the compass at *T*, and set the compass radius to $\overline{TU}$. Draw an arc as shown.
   c. Keeping the same radius, center the compass at *U* and draw another arc intersecting the first one. Let *V* be the point of intersection.
   d. Draw $\overline{TV}$ and $\overline{UV}$.

   e. Explain how you know the triangle is an equilateral triangle, and each of its interior angles has a measure of 60°.

Geometric Constructions, Part 2 (pp. 3 of 7)

3. Given a circle, its center point, and a point on the exterior of the circle, constructs a line through the exterior point, tangent to the circle.
   a. Begin with a circle with center at *C*.
   b. *P* is on the exterior of the circle.
   c. Draw line segment $\overline{CP}$, and construct *M*, the midpoint of $\overline{CP}$. **Refer to the perpendicular bisector construction for the construction of the midpoint.**
   d. Center the compass on *M*. Draw a circle through *C* and *P*. It will intersect the other circle at two points, *R* and *S*.
   e. *R* and *S* are the tangent points. $\overleftrightarrow{PR}$ and $\overleftrightarrow{PS}$ are tangent to the circle centered on *C*.

Geometric Constructions, Part 2 **(pp. 4 of 7)**

4. Construct the center point of a given circle.
   a. Begin with a circle, but no center point.
   b. Draw chord $\overline{AB}$.
   c. Construct the perpendicular bisector of chord $\overline{AB}$. Let C and D be the points where it intersects the circle. **Refer to the construction of a perpendicular bisector.**
   d. Chord $\overline{CD}$ is a diameter of the circle. Construct P, the midpoint of diameter $\overline{CD}$. P is the center point of the circle. **Refer to the construction of a perpendicular bisector.**

Geometric Constructions, Part 2 (pp. 5 of 7) KEY

5. Given a triangle, circumscribe a circle.
   a. Begin with ΔSTU.
   b. Construct the perpendicular bisectors of $\overline{ST}$ and $\overline{TU}$. **Refer to the construction of a perpendicular bisector.** Label the point of intersection as *P*.
   c. Center the compass on *P*, and draw the circle through *S*, *T*, and *U*.

Geometric Constructions, Part 2 (pp. 6 of 7)

6. Given a triangle, inscribe a circle.
   a. Begin with ΔKLM .
   b. Construct the bisectors of $\angle K$ and $\angle L$. **Refer to the construction of an angle bisector.** Let *Q* be the intersection of the two angle bisectors.
   c. Construct a line through *Q*, perpendicular to $\overline{KL}$. Let *R* be the point of intersection. **Refer to the construction of a perpendicular through a given point.**
   d. Center the compass on *Q*, and draw a circle through *R*. The circle will be tangent to all three sides of a triangle.

Geometric Constructions, Part 2 (pp. 7 of 7)

7. Divide a line segment into n congruent line segments. In this example, n = 5.
   a. Begin with $\overline{AB}$. It will be divided into five congruent line segments.
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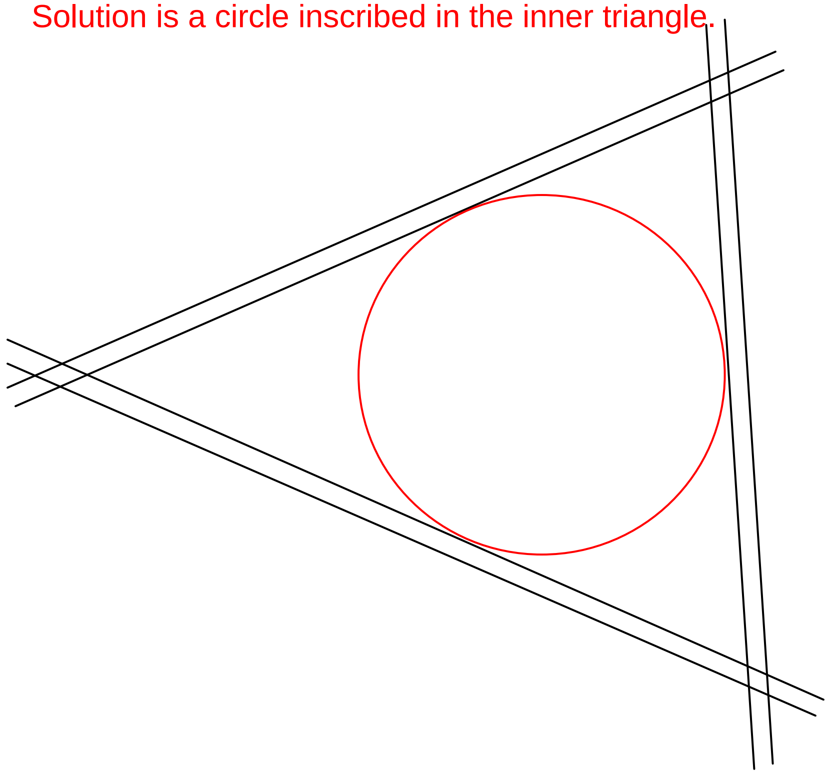
   c. Draw an arc with the compass centered at point *A*, with radius *BC*. Draw a second arc with the compass centered at point *B*, with radius *AC*. Label the intersection point *D*. Note that *ACBD* is a parallelogram.
   d. Use the compass to step off points along $\overline{DB}$, using the same radius that was used for the points along $\overline{AC}$.
   e. Use the straightedge to connect the corresponding points. These line segments will be parallel. They cut $\overline{AC}$ and $\overline{DB}$ into congruent segments. Therefore, they must also cut $\overline{AB}$ into congruent segments.

Applications of Constructions (pp. 1 of 3) KEY

1. Grant Park is putting a sprinkler system in all sections of the park. One section is in the shape of a triangle with a sidewalk all the way around. The park is hiring Ernie's Irrigation to put in the sprinkler system. They want to make sure that water is not wasted by watering the sidewalks. Use constructions to locate where to place the circular sprinkler head so that the water will reach just to the sidewalk on all three sides of the triangular section of the park.

   a. How would you locate the point at which to place the sprinkler head? Explain your reasoning.
   Bisect two of the angles of the park. Construct a perpendicular to one of the sides through the point of intersection of the two angle bisectors. Inscribe a circle inside the triangular section of the park using the intersection of the bisectors as the center of the circle.

   b. Use a compass and straightedge to **construct** the location of the sprinkler head on the diagram of the park section.
   Solution is a circle inscribed in the inner triangle.

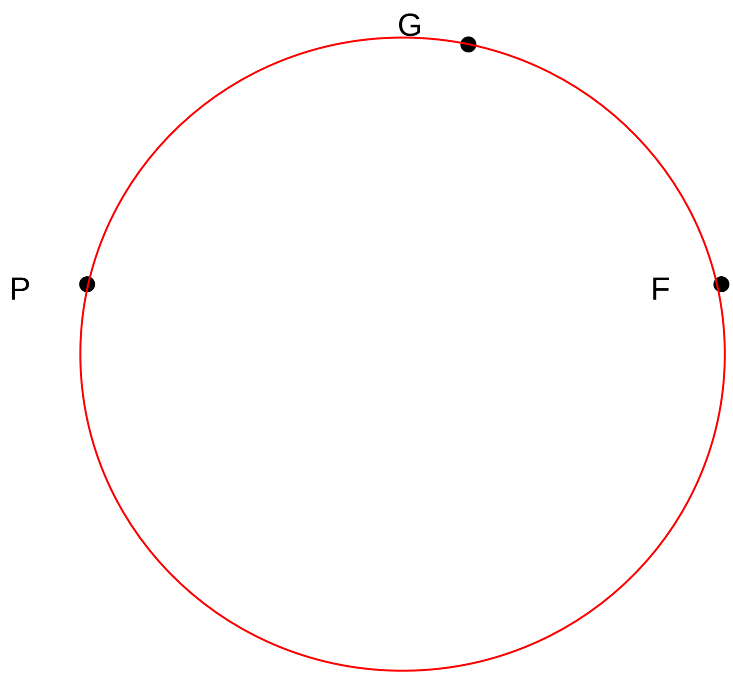


Applications of Constructions (pp. 2 of 3) KEY

2. The local city park wants to open a refreshment stand that is located equidistant from all three of the most popular attractions in the park: the rose garden, the reflecting pool, and the fountain.

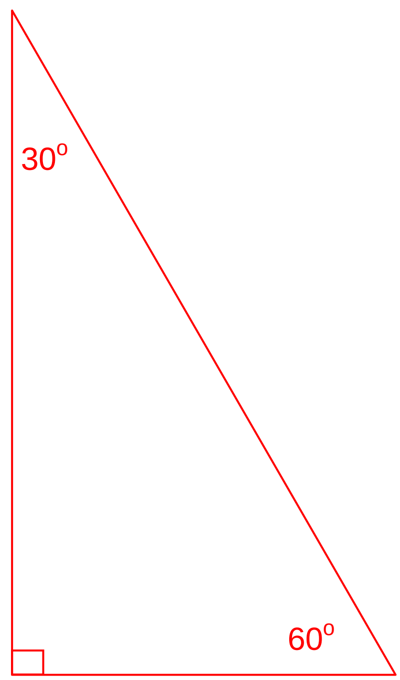
   a. How would you locate the point at which to place the refreshment stand? Explain your reasoning.
   Construct two segments using the three points. Find the perpendicular bisector for each segment. The point of intersection of the perpendicular bisectors is the center of a circle with points G, P and F on the circle. The center of the circle is the location of the refreshment stand that is equidistant from the rose garden, the reflecting pool, and the fountain.

   b. In the diagram below, point G is the rose garden, point P is the reflecting pool, and point F is the fountain. Use a compass and straightedge to **construct** the location of the refreshment stand in the diagram below.
   Solution is a circle passing through the three points.



Applications of Constructions (pp. 3 of 3) KEY

3. Use your knowledge of geometry and special right triangles to construct a 30°-60°-90° triangle.
Student methods will vary. One possible method is to construct the perpendicular bisector of a segment, and then construct the hypotenuse of the triangle so that it is as long as the original segment, thus ensuring the 30°-60°-90° ratio. An alternate method is to construct an equilateral (and thus equiangular) triangle, and then bisect one of the angles to create the 30°-60°-90° triangle.



Applications of Constructions (pp. 1 of 3)

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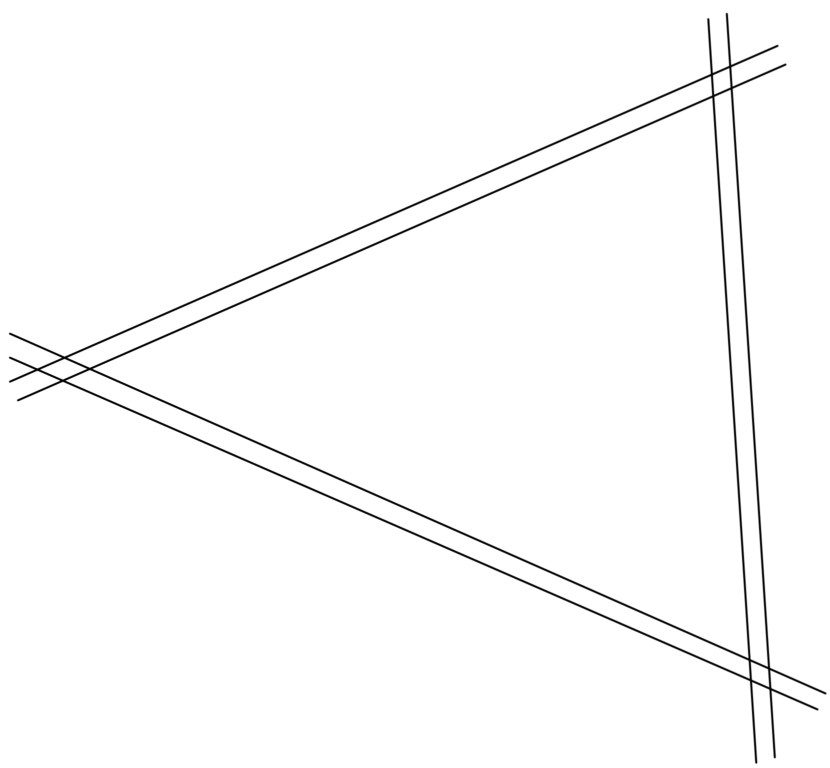
    a. How would you locate the point at which to place the sprinkler head? Explain your reasoning.

    b. Use a compass and straightedge to **construct** the location of the sprinkler head on the diagram of the park section.

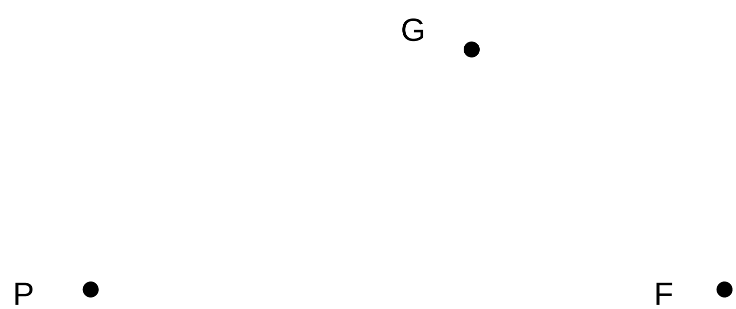


Applications of Constructions (pp. 2 of 3)

2. The local city park wants to open a refreshment stand that is located equidistant from all three of the most popular attractions in the park: the rose garden, the reflecting pool, and the fountain.

   a. How would you locate the point at which to place the refreshment stand? Explain your reasoning.

   b. In the diagram below, point G is the rose garden, point P is the reflecting pool, and point F is the fountain. Use a compass and straightedge to **construct** the location of the refreshment stand in the diagram below.



Applications of Constructions (pp. 3 of 3)

3. Use your knowledge of geometry and special right triangles to construct a 30°-60°-90° triangle.

Take Me Out to the Ballpark! KEY

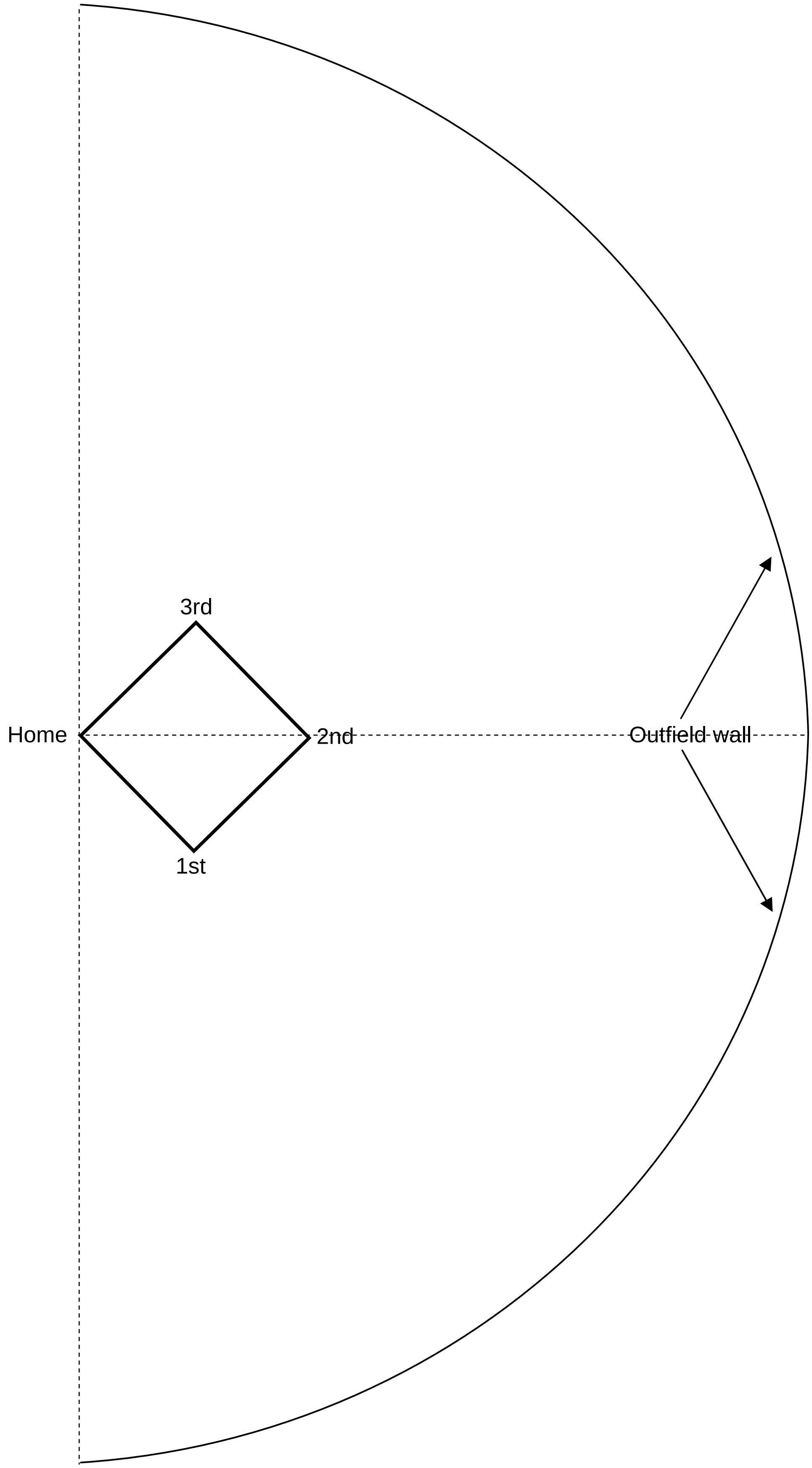
In the beginning of this lesson, you examined a method for laying out a square baseball diamond using measurement. In this evaluation, you will create a layout of a baseball field using geometric constructions.

- Use the given line segment to represent the length of the baselines.

- Construct the square baseball infield.
- Locate and label each base (1^{st}, 2^{nd}, 3^{rd}, and home).
- Locate and label the center of the infield which is the pitcher's mound.
- Locate and label the midpoint of the baseline between second and third base which is the approximate location of the shortstop.
- Mark the outfield wall which has a radius of three times the length of a baseline from home plate.
- You may construct your layout on the back of this sheet.
- Include a brief step-by-step summary of how you constructed your field in the space below.

Student methods will vary. The teacher must study the students' work in order to discern if the method outlined in the students' step-by-step summary produces the baseball field layout as outlined above. See Sample diagram on next page.

Take Me Out to the Ballpark Sample Diagram



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